Regular Grammars

Lecture 12
Section 3.3

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Outline

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Regular Grammars

Definition (Regular grammar)

A regular grammar is a 4-tuple (V, T, S, P), where

- V is a set of symbols, called variables, or nonterminals.
- T is a set of symbols, called terminals.
- $S \in V$ is the start symbol.
- P is a set of production rules, or rewrite rules of the following forms:
  - $A \to aB$
  - $A \to \lambda$

  where A and B are nonterminals and a is a terminal.

Regular Grammars

- To generate strings from the grammar, we begin with the start symbol and apply the production rules until we obtain a string of all terminals.

Example

Example (Regular grammar)

Let the rules be

$$\begin{array}{ll} S \to \mathbf{a}X & Y \to \mathbf{a}Y \\ S \to \mathbf{b}Y & Y \to \mathbf{b}Y \\ S \to \lambda & Y \to \lambda \end{array}$$

$$\begin{array}{ll} X \to \mathbf{a}S & Z \to \mathbf{a}Y \\ X \to \mathbf{b}Z & Z \to \mathbf{b}X \\ X \to \lambda & \end{array}$$

Example

Example (Regular grammar)

This list of rules may be consolidated as

$$
\begin{aligned}
S &\to \mathbf{a}X \mid \mathbf{b}Y \mid \lambda \\
X &\to \mathbf{a}S \mid \mathbf{b}Z \mid \lambda \\
Y &\to \mathbf{a}Y \mid \mathbf{b}Y \mid \lambda \\
Z &\to \mathbf{a}Y \mid \mathbf{b}X
\end{aligned}
$$

Example

Example (Regular grammar)

- What strings can be obtained by these rules?
  - $S \Rightarrow \mathbf{a}X \Rightarrow \mathbf{aa}S \Rightarrow \mathbf{aab}Y \Rightarrow \mathbf{aab}$.
  - $S \Rightarrow \mathbf{b}Y \Rightarrow \mathbf{bb}Y \Rightarrow \mathbf{bba}Y \Rightarrow \mathbf{bba}$.
- What other strings?
- Is there a pattern?

The Language of a Grammar

Definition (Derivation)

A derivation is a sequence of applications of production rules, beginning with the start symbol and ending with a string in Σ^*.

Definition (Language of a grammar)

The language of a grammar is the set of all strings in Σ^* that can be derived from the grammar.

Outline

Regular Grammars and Regular Languages

Theorem (Equivalence of regular grammars and regular languages)

A language is regular if and only if it is the language of a regular grammar.

Proof ($\Leftarrow$).

- Given a DFA $M = (Q, \Sigma, \delta, q_0, F)$,
  - Let $V = Q$.
  - Let $T = \Sigma$.
  - Let $S = q_0$.
  - For each transition $\delta(p, a) = q$, write a production $p \to aq$.
  - For each $q \in F$, write a production $q \to \lambda$.
- It is clear that the strings derived from this grammar are exactly the strings in $L(M)$.

□

Regular Grammars and Regular Languages

Proof ($\Rightarrow$).

- All the steps in the previous proof are reversible.

□

Example

Example (Constructing a DFA from a regular grammar)

- Construct a DFA from the grammar

$$\begin{aligned} S &\to \mathbf{a}X \mid \mathbf{b}Y \mid \lambda \\ X &\to \mathbf{a}S \mid \mathbf{b}Z \mid \lambda \\ Y &\to \mathbf{a}Y \mid \mathbf{b}Y \mid \lambda \\ Z &\to \mathbf{a}Y \mid \mathbf{b}X \end{aligned}$$

- What is the language of the grammar?

Example

Example (Constructing a regular grammar from a DFA)

- Construct a regular grammar for the regular language

$$\{w \mid w \text{ is a binary number that is a multiple of } 3\}.$$

- Write a regular expression for the language in the previous example.

Example

Example (Constructing a DFA from a nonregular grammar)

- Construct a DFA from the following grammar.

$$S \to \mathbf{aa}S \mid \mathbf{abb}A$$
$$A \to S \mid \mathbf{b}S \mid \mathbf{ba}.$$

Outline

Assignment

Assignment

- Section 3.3 Exercises 1, 2, 3, 4, 5, 10, 11, 12.